Observation of Chaos in a Microwave Limiter Circuit

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Abstract—**Chaotic behavior has been observed in a microwave limiter circuit that consists of a shunt p-i-n diode cascaded with a 0.68-dB/MHz rolloff bandpass filter. The chaos manifests itself as a broad-band power spectral density of the output signal. The chaos appears and disappears as the incident power to the limiter circuit is increased, suggesting bifurcations and periodic windows in the chaotic region. Data are included that illustrate classic characteristics of chaos such as period doubling and broad-band spectra.**

I. Introduction

CHAOS IS A complex oscillation that occurs in nonlinear systems. Chaos is known to occur in a diverse range of physical devices [1] such as electronic circuits, lasers, and digital filters, as well as in many mechanical systems [2]. Chaos can occur in very simple circuits; in fact, chaos can occur in a driven RLC circuit [3] with a nonlinear diode. Chaos in diode circuits is a textbook example of chaotic behavior.

It is useful to describe a chaotic circuit in terms of its state-space coordinates. A lumped driven RLC diode circuit has at least three independent state coordinates, which can be taken as the drive phase, capacitor voltage, and inductor current. The state-space coordinates uniquely determine the state of the system at any given time. A point in state-space thus determines the state of the system, and the motion of this point traces out a curve called a "state-space trajectory."

Fig. 1 shows the limiter circuit with the cover removed to show the internal structure. The circuit is a combination of a five-element comb-line filter with a p-i-n limiter diode in shunt with the last resonator element. The filter provides a sharply defined frequency response having a 3-dB bandwidth of 52 MHz centered at 1.575 GHz.

The circuit was initially designed as a protection device for communications systems [5]. In the course of experimental characterization, the output signal began to bifurcate into subharmonics as the input signal power was increased (period doubling). Fig. 2(a) shows a state-space portrait, and Fig. 2(b) shows the power spectrum for a period-two bifurcation. This frequency division continued until the frequency spectrum was continuous—an indication of the onset of chaos. As the input power was increased further, windows of periodic and chaotic motion appeared alternately.

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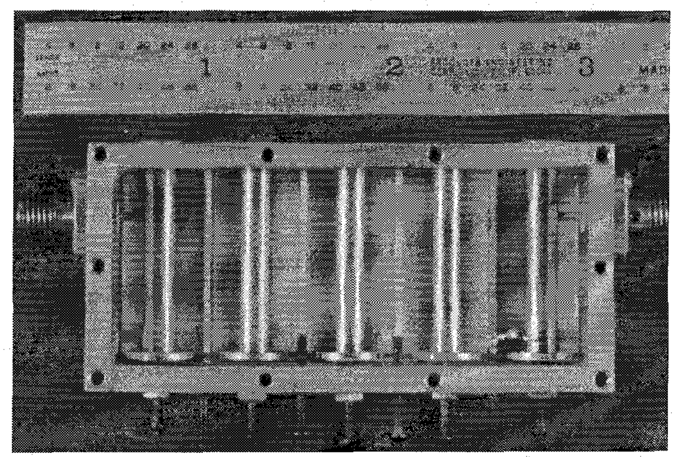


Fig. 1. Limiter circuit realized by cylindrical cavity resonators. (Note diode at right-most resonator position.)

II. Chaotic Behavior

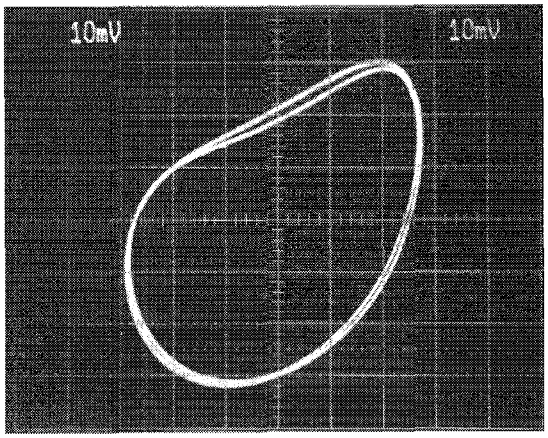
An experiment was devised to measure the frequency spectrum of the output signal and generate state-space *portraits* as the input signal power was varied. State-space portraits are generated by use of a principle called time-delay embedding [6], which allows one to expose n-dimensional projections of the attractor by displaying the output signal, $V(t)$, versus time-delayed versions of itself, $V(t-\delta t), V(t-2\delta t), \ldots, V(t-n\delta t)$ ($\nu\delta t$ is on the order of $\lambda/4$ of the input signal, where λ is the wavelength and ν is the velocity in the transmission line).

Chaotic behavior was usually made evident by an extremely broad-band spectral content, as well as an unresolved, yet bounded, state-space portrait (see Fig. 3). Individual trajectories are not clearly distinguished because we are operating at the extreme edge of the oscilloscope bandwidth. The output spectral content shows the dispersal of the input signal in to a greater than 1-GHz-wide distribution.

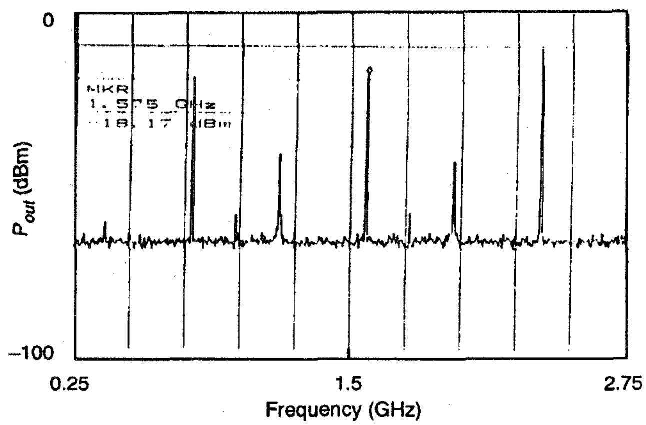
There were several instances of the above-mentioned period doubling, several instances of high-dimensional chaotic motion—regions where periodic and chaotic motion occurred intermittently—and even some regions where there existed multiply periodic motion (on a *torus* instate-space). Fig. 4 shows state-space portrait examples for both motion on a torus and intermittent behavior. The intermittent state shown in Fig. 4(b) is low-frequency intermittency between period-2 (the clearly resolved region) and chaotic motion (the unresolved region).

III. Conclusion

We have observed experimentally that a simple driven microwave circuit can produce highly complex behavior. Vari-

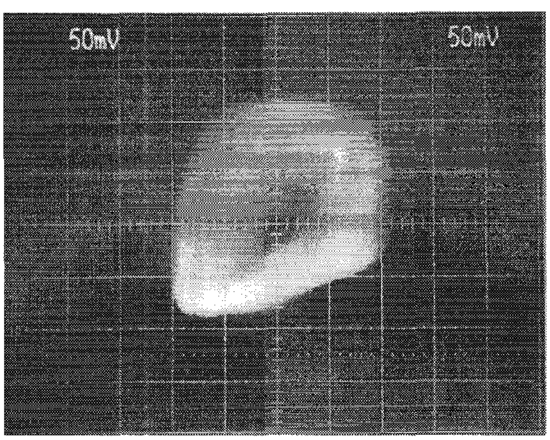


(a)



(b)

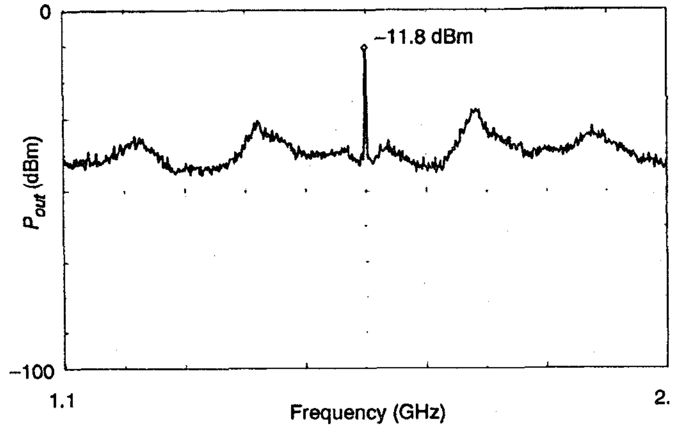
Fig. 2. Period 2: (a) State-space portrait and (b) spectrum analyzer plot.



(a)

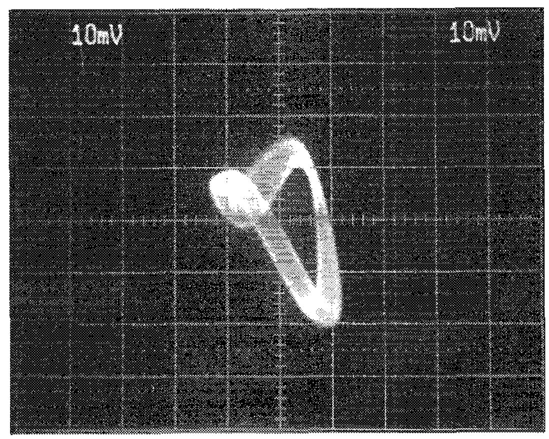
Fig. 3. (a) State-space portrait.

ation of a system parameter (such as drive frequency or input power) produced several periodic, aperiodic, and chaotic attractors. We were convinced that it was the combination of a filter and a nonlinear element (limiter) that provided the proper parameter conditions for chaos to exist. We experimentally supported this assertion by observing the behavior of several off "of the shelf" filters and limiters in combination. Similar behavior resulted.

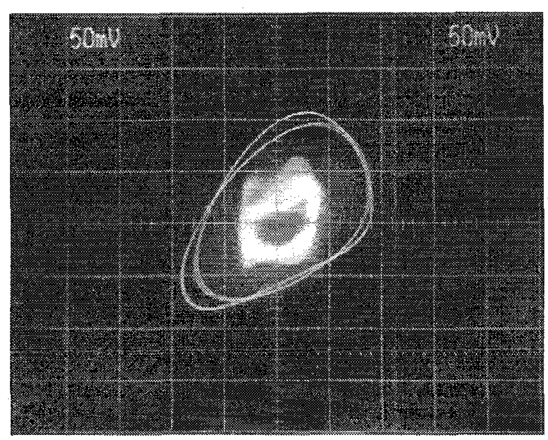


(b)

Fig. 3. (b) Frequency spectra for the limiter circuit driven into chaos ($P_{in} =$ 37 mW).



(a)



(b)

Fig. 4. Examples of (a) toroidal motion and (b) an intermittent state.

Because of the specific use of the circuit, this type of complex behavior is undesirable. However, it is possible to use the complexity and extreme sensitivity to disturbances that chaos offers for engineering purposes in microwave systems. It has recently been demonstrated [7] that a single chaotic oscillator can be controlled [8] to produce complex wave forms (even digital communication signals), and we are pursuing the application of these principles to microwave sources (synchronization [9] of chaotic oscillations is also possible). These types of control can be carried out using tiny control perturbations, thus significantly reducing complexity and power requirements in systems.

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